SySc 512 – Quantitative Methods of Systems Science

MIDTERM EXAM: FEBRUARY 9, 2006

(1) A gradient system has the following potential function:

$$V(x, y) = x^2 + yx + y^2$$

(a) Write the differential equations that describe the associated gradient system.
(b) Find the fixed points.
(c) Sketch the nullclines on graph paper.
(d) Draw flow arrows on the nullclines.
(e) Compute the Jacobian at the fixed points.
(f) Write the characteristic equation for the eigenvalues at the fixed points.
(g) What are the eigenvalues at the fixed points?
(Hint: roots of $ax^2 + bx + c = 0$ are $(-b \pm \sqrt{b^2 - 4ac})/2a$)
(h) Are the fixed points stable? Why?

(2) Consider the following dynamical system:

$$\begin{aligned} \frac{dx}{dt} &= -x(2 - x) - 2y \\ \frac{dy}{dt} &= \frac{x}{2} - y \end{aligned}$$

(a) Is this a gradient system? If so, what is the potential function?
(b) Find the fixed points.
(c) Sketch the nullclines on graph paper.
(d) Draw flow arrows on the nullclines.
(e) Choose a fixed point and compute the Jacobian at that fixed points.
(f) Write the characteristic equation for the eigenvalues at your chosen fixed point.
(g) What are the eigenvalues at your chosen fixed point?
(Hint: roots of $ax^2 + bx + c = 0$ are $(-b \pm \sqrt{b^2 - 4ac})/2a$)
(h) Is your chosen fixed point stable? Why?