SySc 512 Midterm Previews

1) A) Dyn Eqs:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = -\nabla V(x,y) = \begin{bmatrix}-\frac{\partial}{\partial x}V\\ -\frac{\partial}{\partial y}V\end{bmatrix} = \begin{bmatrix}-(2x+y)\\ -(x+2y)\end{bmatrix}$$

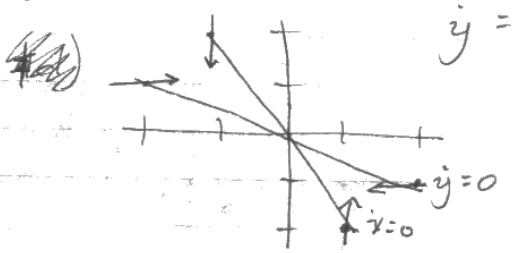
B) Fixed pts:

$$\left.\begin{aligned}\dot{x}=0\\ \dot{y}=0\end{aligned}\right\} \Rightarrow \left.\begin{aligned}-2x=y\\ -2y=x\end{aligned}\right\} \Rightarrow -4x=0 \Rightarrow x=0$$

$$\boxed{(\bar{x},\bar{y})=(0,0)}$$

c) Nullclines: $\dot{x}=0 \Rightarrow y=-2x$

$\dot{y}=0 \Rightarrow y=-x/2$



d) $@(x,y)=(1,-2) \Rightarrow \dot{y}>0$

$@(x,y)=(2,-1) \Rightarrow \dot{x}<0$

e) $J = \begin{bmatrix}\frac{\partial}{\partial x}F_x & \frac{\partial}{\partial y}F_x\\ \frac{\partial}{\partial x}F_y & \frac{\partial}{\partial y}F_y\end{bmatrix} = \begin{bmatrix}-2 & -1\\ -1 & -2\end{bmatrix}$

Or, $V(x,y) = -\frac{1}{2}\vec{x}^T J \vec{x} = \frac{1}{2}\begin{bmatrix}x & y\end{bmatrix}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$

f) $(-2-\lambda)(-2-\lambda)-1=0$

$\lambda^2+4\lambda+4-1=0 \Rightarrow \frac{-4\pm\sqrt{16-4\cdot 3}}{2} = -2\pm 1$

g)

h) Stable because both eigenvalues <0.

5. Sc 512, Midterm Preview (cont.)

2) a) Is $\frac{\partial}{\partial x} F_y - \frac{\partial}{\partial y} F_x = 0$?

$2x F_y = \frac{1}{2}$, $2y F_x = -2$ => not gradient

b) $\dot{x} = 0$, $\dot{y} = 0$ => $-x(2-x) - 2y = 0$, $\frac{x}{2} - y = 0$ => $y = -\frac{1}{2}x(2-x)$, $y = x/2$

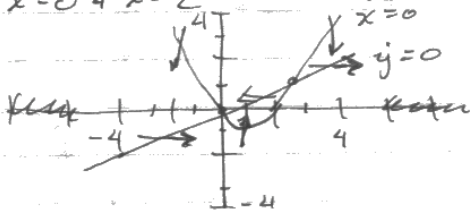
$\frac{x}{2} = -\frac{x}{2}(2-x)$, if $x = 0$ => $y = 0$, $(\bar{x}_1, \bar{y}_1) = (0,0)$

$-1 = (2-x)$ if $x = 3$ => $y = \frac{3}{2}$, $(\bar{x}_2, \bar{y}_2) = (3, \frac{3}{2})$

c) $\dot{x} = 0$ => $y = \frac{1}{2}x^2 - x$

w/ zeros @ $x = 0$ & $x = 2$

$\dot{y} = 0$ => $y = x/2$



d) $\dot{x}=0$) if $y > x/2 \Rightarrow \dot{y} < 0$

if $y < x/2 \Rightarrow \dot{y} > 0$

$\dot{y}=0$) if $y > \frac{1}{2}x^2 - 2x \Rightarrow \dot{x} < 0$

if $y < \frac{1}{2}x^2 - 2x \Rightarrow \dot{x} > 0$

e) $J = \begin{bmatrix} \frac{\partial}{\partial x} F_x & \frac{\partial}{\partial y} F_x \\ \frac{\partial}{\partial x} F_y & \frac{\partial}{\partial y} F_y \end{bmatrix} = \begin{bmatrix} 2x-2 & -2 \\ \frac{1}{2} & -1 \end{bmatrix}$

@ (0,0), $J = \begin{bmatrix} -2 & -2 \\ \frac{1}{2} & -1 \end{bmatrix}$

f) $(-2-\lambda)(-1-\lambda) + 1 = 0$

$\Rightarrow \lambda^2 + 3\lambda + 3 = 0$

g) $\lambda = \frac{-3 \pm \sqrt{9 - 4 \cdot 3}}{2}$

$= -\frac{1}{2}(3 \pm i\sqrt{3})$

h) Stable because $Re(\lambda_\pm) < 0$

@ $(3, \frac{3}{2})$, $J = \begin{bmatrix} 4 & -2 \\ \frac{1}{2} & -1 \end{bmatrix}$

$(4-\lambda)(-1-\lambda) + 1 = 0$

$\Rightarrow \lambda^2 - 3\lambda - 3 = 0$

$\lambda = \frac{3 \pm \sqrt{9 + 12}}{2}$

$= \frac{1}{2}(3 \pm \sqrt{21})$

Unstable because $\frac{1}{2}(3 + \sqrt{21}) > 0$