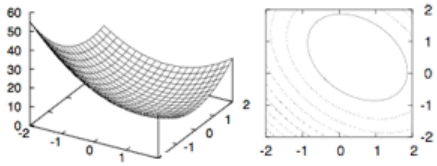
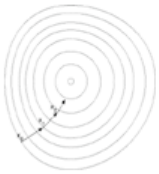
Part 2: Optimization

Apr 25 (07) **Optimization overview**.

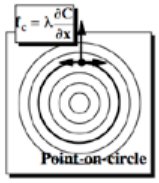


Apr 30 (08) **Dynamics of Optimization**.
(Practice midterm)

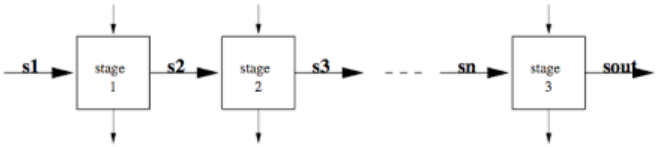


May 2 (09) Midterm.
May 7 (10) Review midterm.

May 9 (11) **Constrained optimization**. HW 5



May 14(12) **Dynamic programming**.



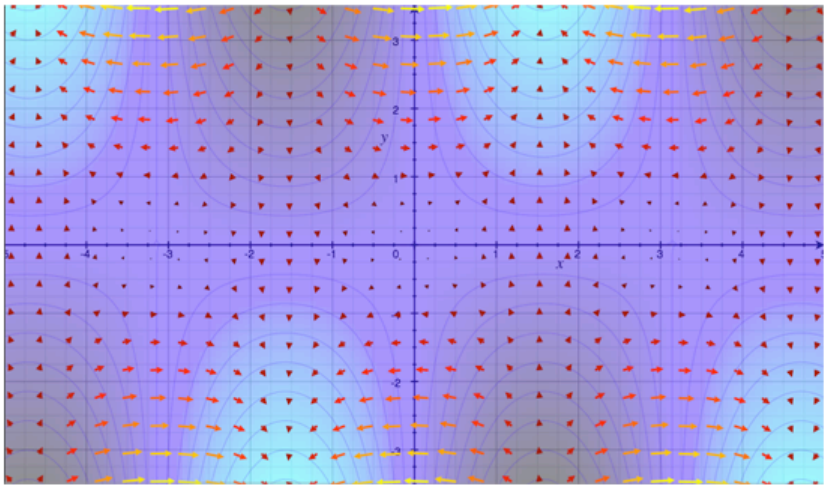
01_Optimize.psd

Gradient Systems: Gradient of a Potential Functions

Vector fields associated to a scalar potential: $V : \mathbb{R}^n \to \mathbb{R}$

$$\frac{d}{dt}\vec{x} = \vec{f}(x) = -\nabla V(x)$$

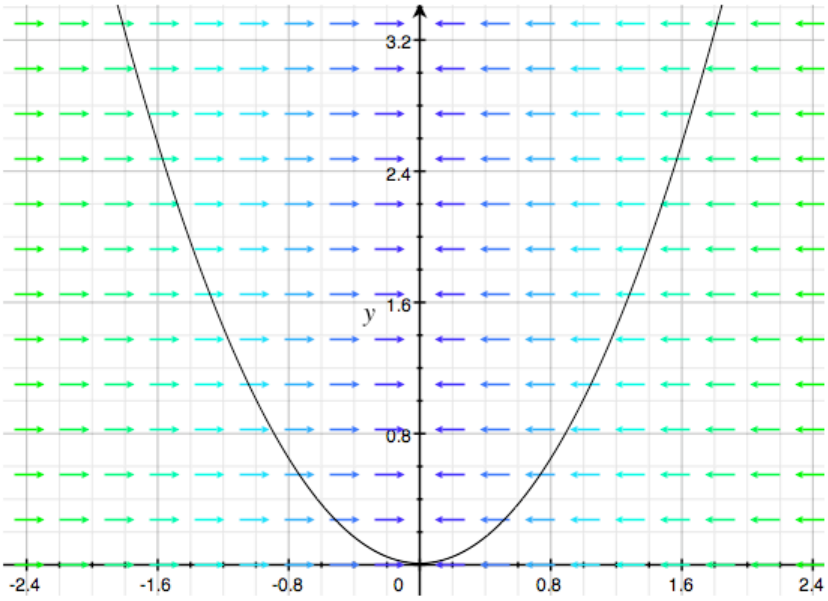
The gradient is the maximal directional derivative: $\nabla V(x) = \begin{bmatrix} \partial/\partial x_1 \\ \partial/\partial x_2 \\ \vdots \\ \partial/\partial x_N \end{bmatrix} V(x)$



$f(x,y)=y\cdot\sin x$

13_WhatGradientSys.psd

Geometry of Gradient Systems: 1-Dimension



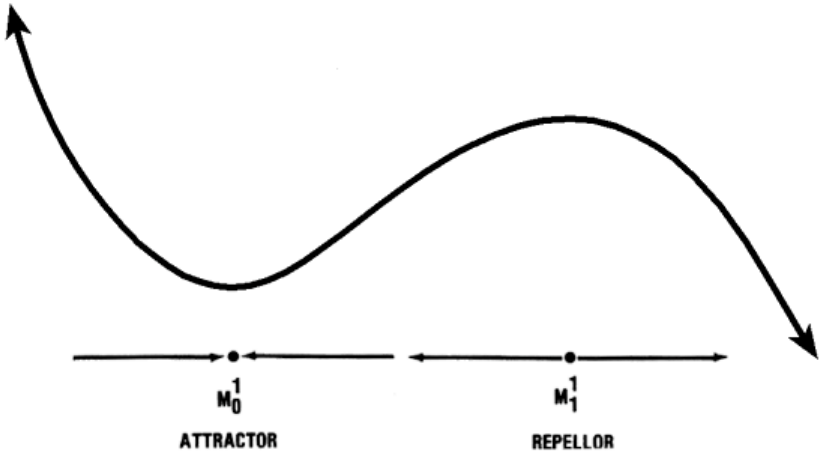
14_1D_levelSets.psd

Attractor versus Repeller: Second Derivative at Fixed Point

The *Hessian* of the potential at each fixed point

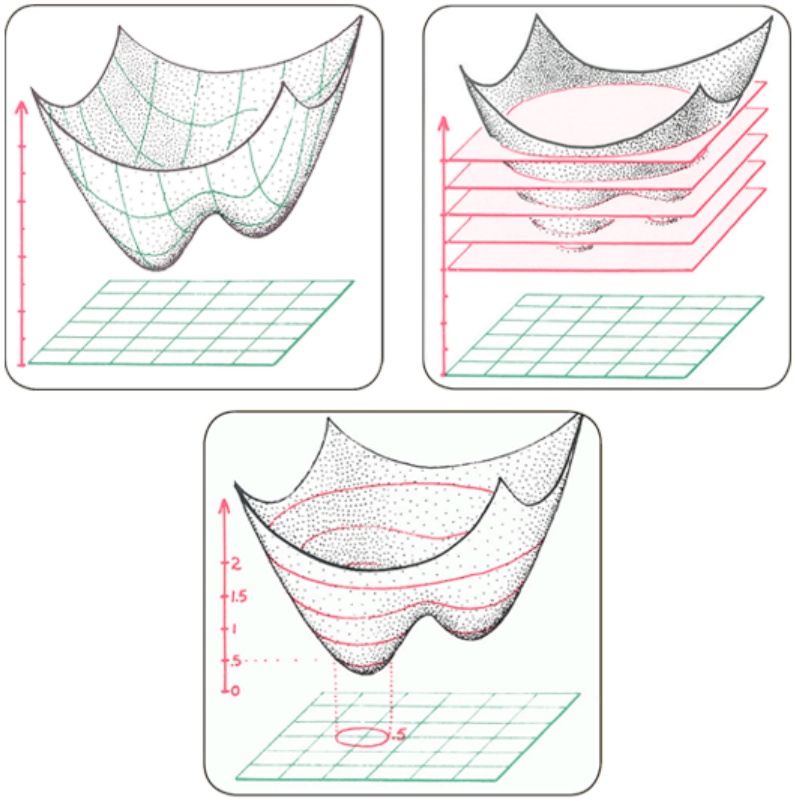
$$H_{i,j}(x) \equiv \frac{\partial^2}{\partial x_i \partial x_j} V(x)\Big|_{x=x_0}$$

determines whether the fixed point is an attractor or repeller:



15_criticalValue.psd

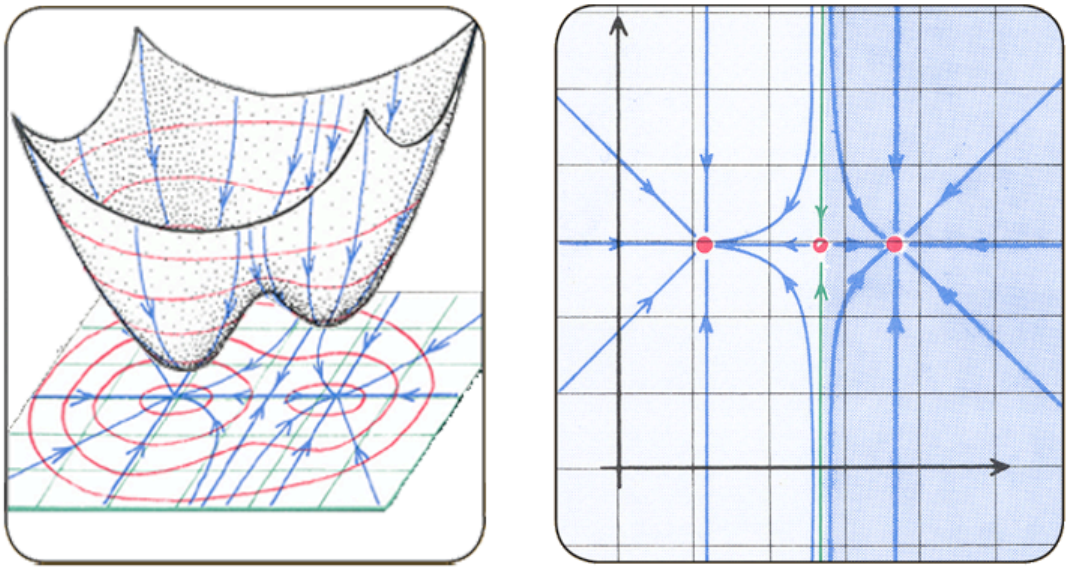
Gradient Systems: Level Sets



Dynamics: The Geometry of Behavior, Ralph Abraham and Chris Shaw (2005)

17_levelSets.psd

Gradient Systems: Forces from Potential Function



Dynamics: The Geometry of Behavior, Ralph Abraham and Chris Shaw (2005)

18_potential.psd