SySc 512 – Quantitative Methods of Systems Science

Homework 4: Optimization.

(1) Verify the first and second order conditions for the local minima and maxima of the following functions:

$$f(x) = x^3(1-x)^2(1+x)$$
$$F(x_1, x_2) = x_1^2 + 2x_2^2 + 2ax_1x_2 + b$$

(2) For the function

$$f(x) = x^3 e^{-x^2}$$

(a) Plot $f(x)$.
(b) Find all values of x for which $f'(x) = 0$.
(c) For each zero $\hat{x}$ of f':
   (i) Give the value of $\hat{x}$.
   (ii) Give the value of $f(\hat{x})$.
   (iii) Determine if $\hat{x}$ is a local minimum, a local maximum, or neither.

You may want to use numerical (computer) methods for parts of this problem.

(3) American Airlines allows carry-on luggage for which the total outside dimensions of each bag is less than 45 inches, ie, $x + y + z < 45''$. What is the largest volume bag that is allowed?

(4) Solve each of the following:

Minimize $\frac{1}{2}(x_1^2 + 2x_1x_2 + 3x_2^2 + 4x_1 + 5x_2)$
Subject to $x_1 - x_2 = 2$

Minimize $x_1^2 + 2x_1x_2 + x_2^2$
Subject to $x_1^2 + 4x_2 = 2$