SySc 512 – Quantitative Methods of Systems Science

Homework 3: Linear Algebra Review. *The purpose of this homework is to give you practice with linear algebra. Exercise 1 must be done by hand, and exercise 2 may be done by hand or you may use a numerical tool like Matlab.*

(1) Perform the calculations each of the following matrices:

$$A_1 = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0.7 & -0.7 \\ 0.7 & 0.7 \end{bmatrix} \quad A_4 = \begin{bmatrix} 0.8 & 1.8 \\ 0 & 1.25 \end{bmatrix}.$$

    (a) For each matrix A calculate:
        (i) Trace.
        (ii) Determinant.
        (iii) Eigenvalues and eigenvectors.

    (b) *Diagonalize* A_4, ie, find matrices Q, Q^{-1}, and D such that $QQ^{-1} = Q^{-1}Q = 1 \equiv \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, D is diagonal, and

$$A_4 = QDQ^{-1}$$

    Explain the relationships amongst the columns of Q, the diagonal elements of D.

(2) Given that

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \\ 3 & -1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & -1 \\ 4 & 2 & 2 \end{bmatrix},$$

    determine the following:
    (a) $A + 4BC$
    (b) The determinant of each matrix.
    (c) The inverse of each matrix if it exists.
    (d) A^3
    (e) The eigenvalues and eigenvectors of B.